Building a Physics Simulator

ESE 6510
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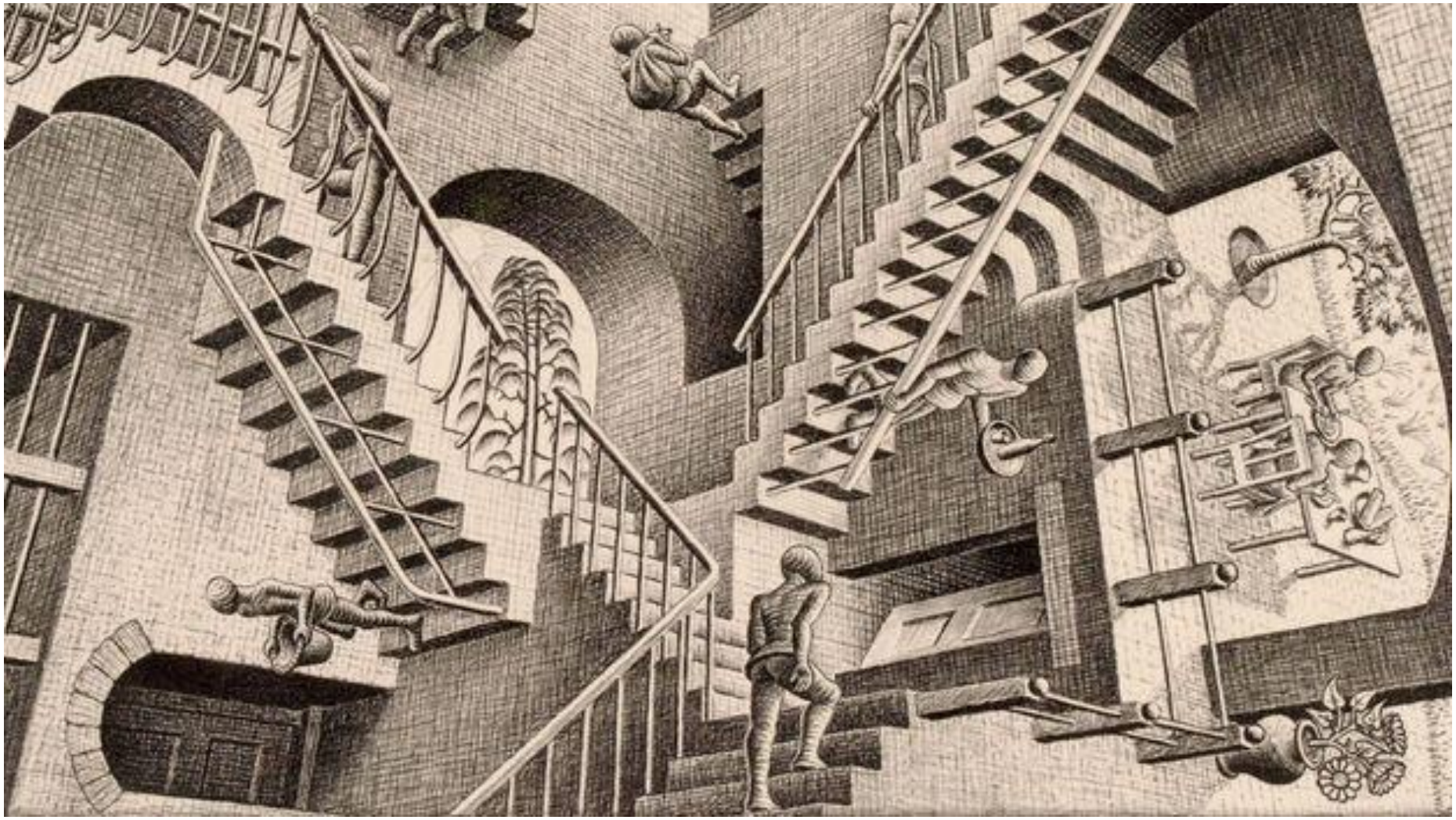
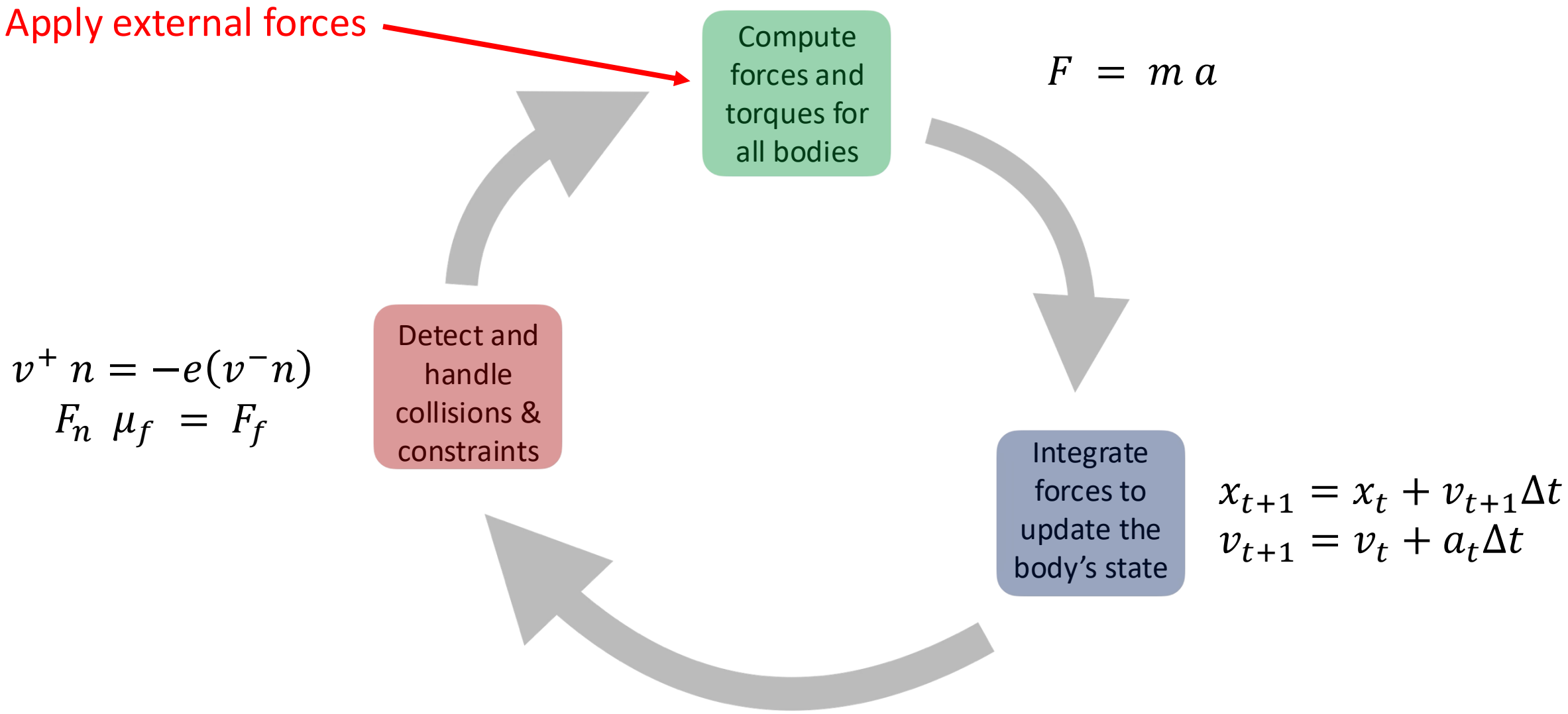


M.C. Escher,

***Relativity*, 1953**

The Anatomy of a Physics Simulator



Dynamics Foundations

Articulated rigid-body motion equations

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau + J^T\lambda$$

Where:

- q : the body's generalized coordinates (e.g., joints + base position)
- $M(q)$: the body's mass/inertia matrix
- $C(q,\ddot{q})$: Coriolis and centrifugal terms
- $G(q)$: Gravity
- τ : applied forces and torques
- $J^T\lambda$: constraint forces/torques (e.g., from contact or joints)

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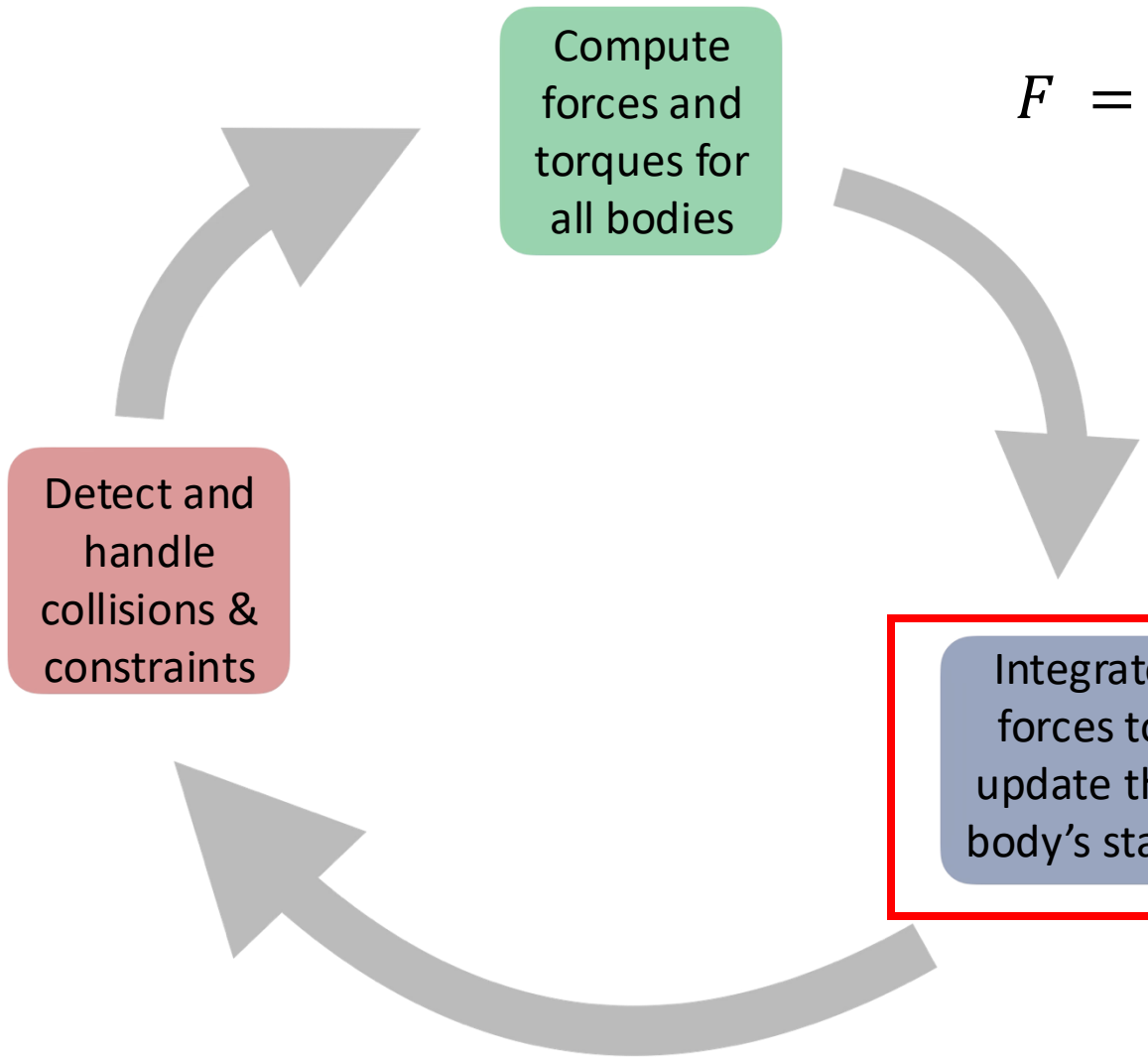


$$F = m\,a$$

$$x_{t+1} = x_t + v_{t+1}\Delta t$$
$$v_{t+1} = v_t + a_t\Delta t$$

$$v^+ n = -e(v^- n)$$
$$F_n\ \mu_f = F_f$$

Numerical Integration

Rigid-body motion equations

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau + J^T \lambda$$

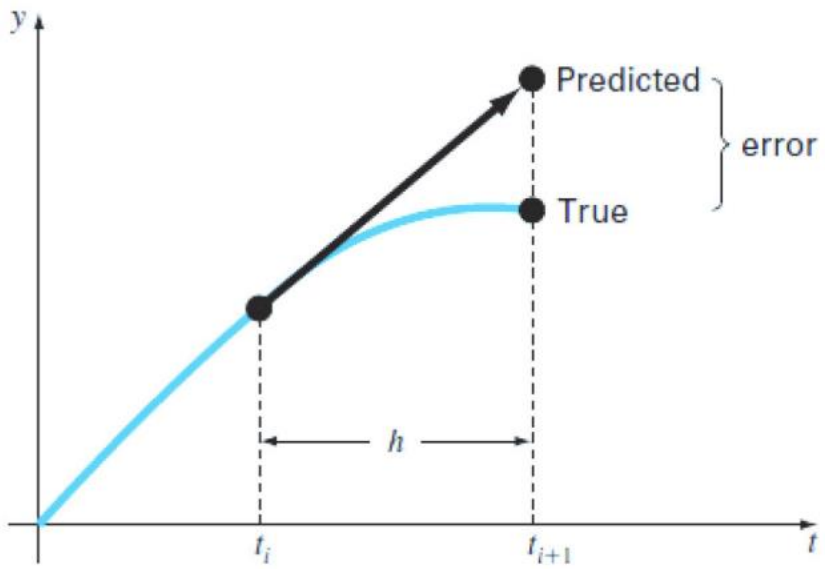
Key insight: We never solve these equations analytically but approximate them numerically step by step.

Example:

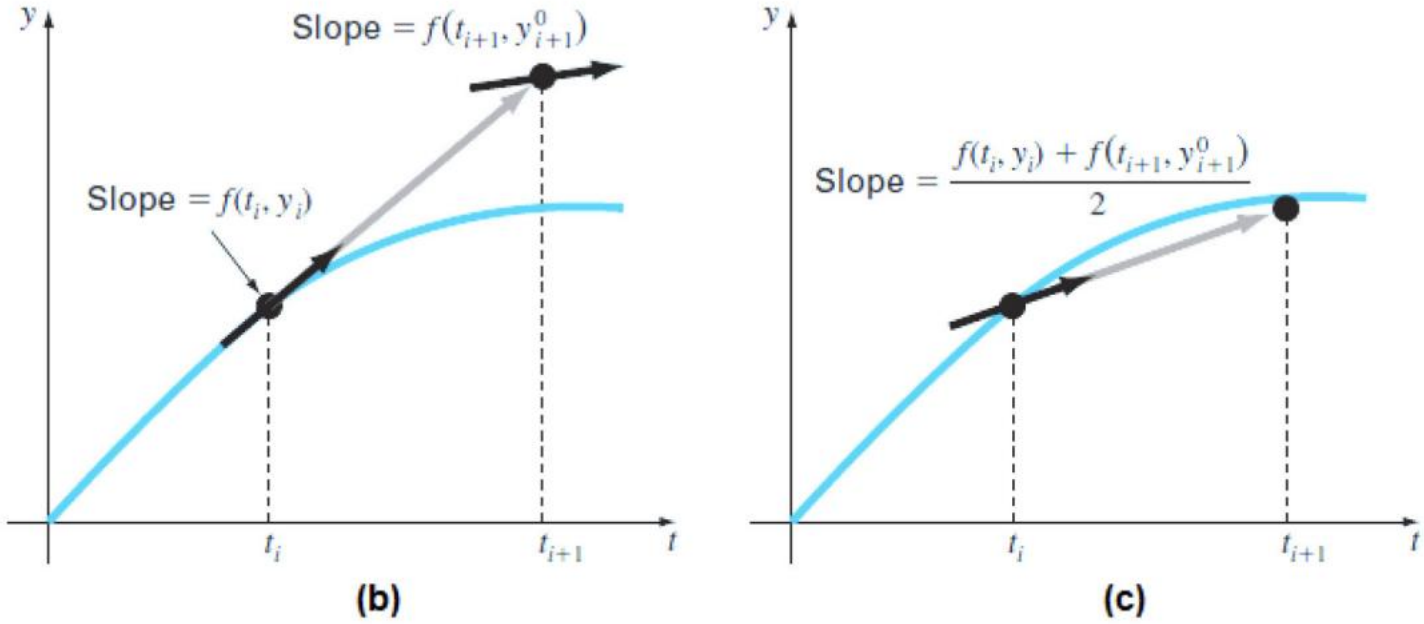
- A point mass falling under gravity:
- $\ddot{y} = -g$

→integrate to get $v(t + dt)$and $y(t + dt)$.

Numerical Integration



Euler's method



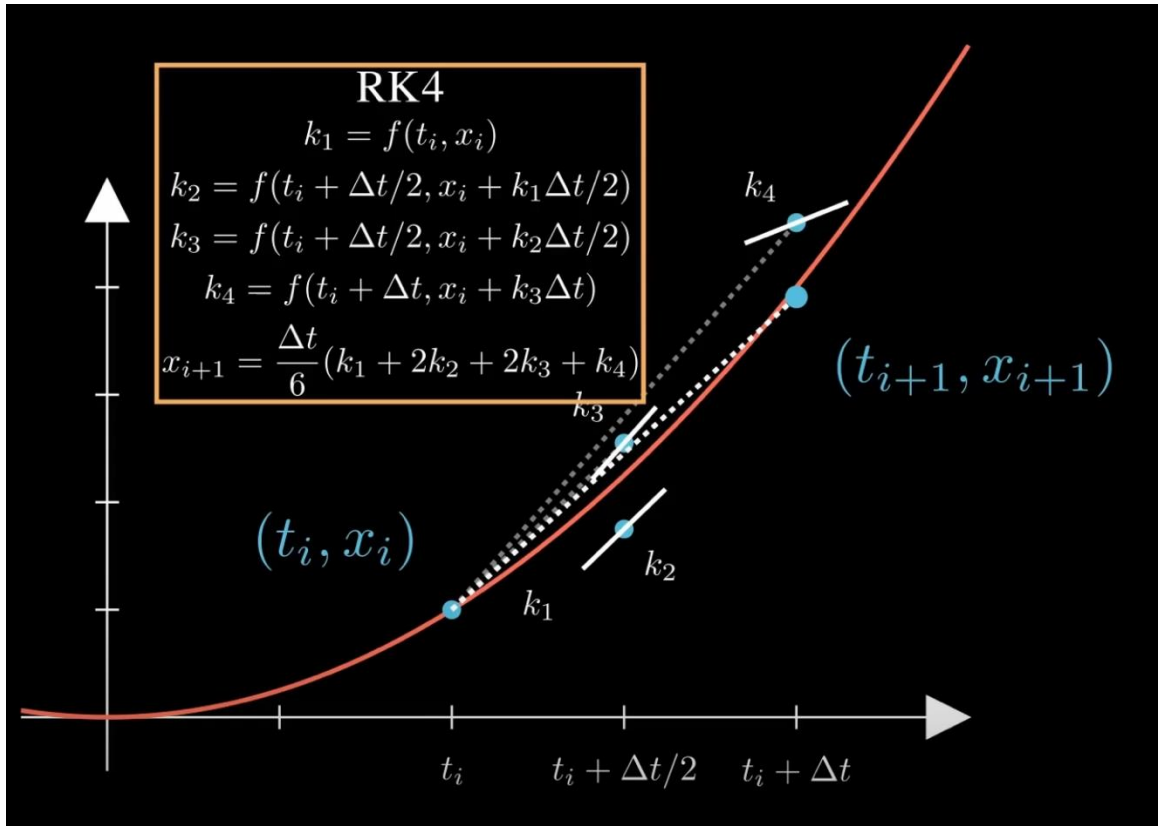
Runge-Kutta second order method (b: predictor, c: corrector)

Most real-time simulators prefer slightly inaccurate but stable integrators: unstable physics looks wrong immediately.

Illustration from Meert et al., 2014

Numerical Integration

Want to know more? This is a good video for a quick overview of the advantages and disadvantages of different integrators:



Source: https://youtu.be/dShtlMl69kY?si=iKDyZIrJ2mpi9Y3h

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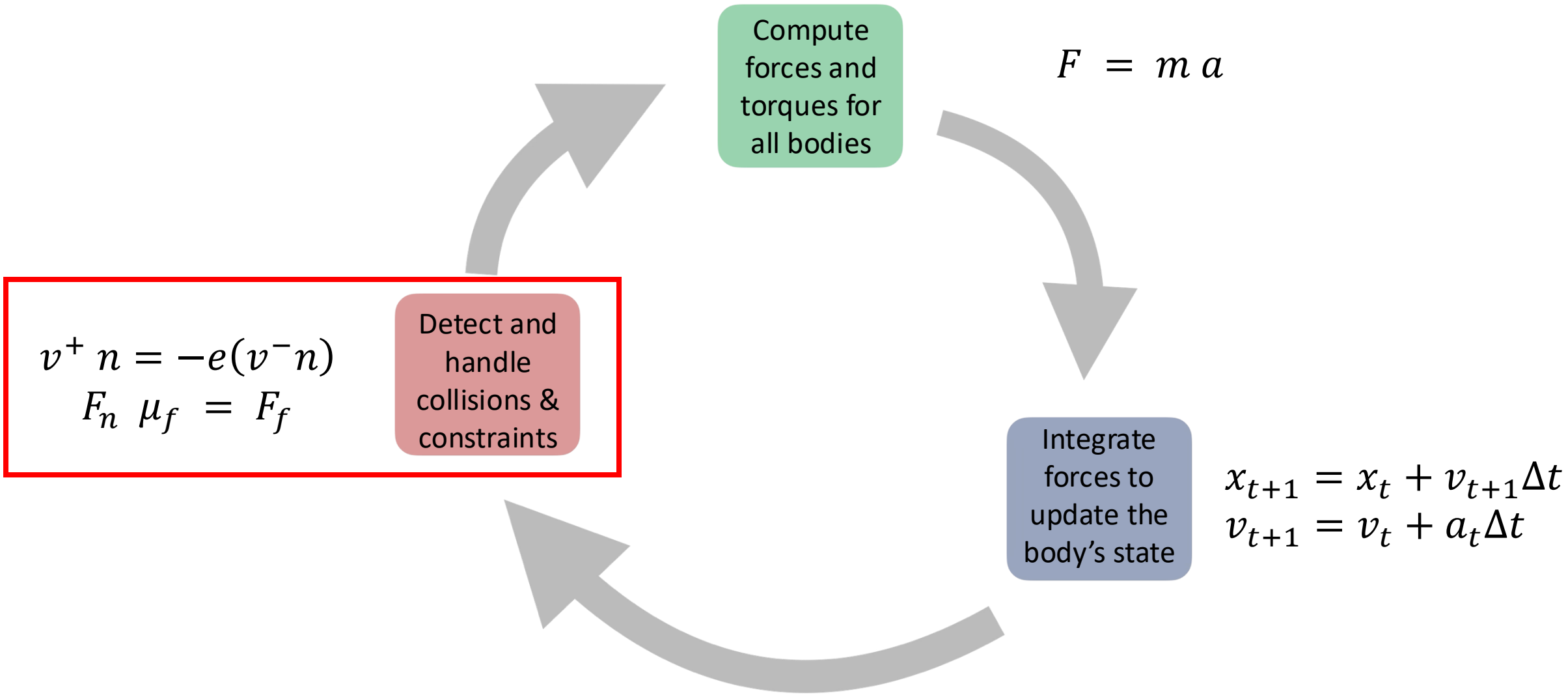

Collision Detection and Response

Two main stages:

1. Collision Detection
   - **Broad Phase:** quickly eliminate pairs that definitely don't collide. Use bounding boxes (AABB) or spatial grids.
   - **Narrow Phase:** compute exact contacts between remaining pairs. Output: contact normal, penetration depth, and contact point.

2. Collision Response
   - **Normal axis**: The bodies that collide do not interpenetrate (e.g., spring-damper system). This results in forces applied to the bodies.
   - **Tangential axes**: Friction forces (Columb's friction cone).

From Collisions to Joints

- Contacts are really just a kind of constraint: objects cannot interpenetrate.

- Generalizing this idea, we can simulate joints: robots joints stay connected and move in specific ways. Examples:

    1. Fixed joints (weld) constrain all motions
    2. Revolute joints only rotate on one axis
    3. Prismatic joints only translate on one axis
    4. … (Cylindrical, Planar, Spherical, Helical)

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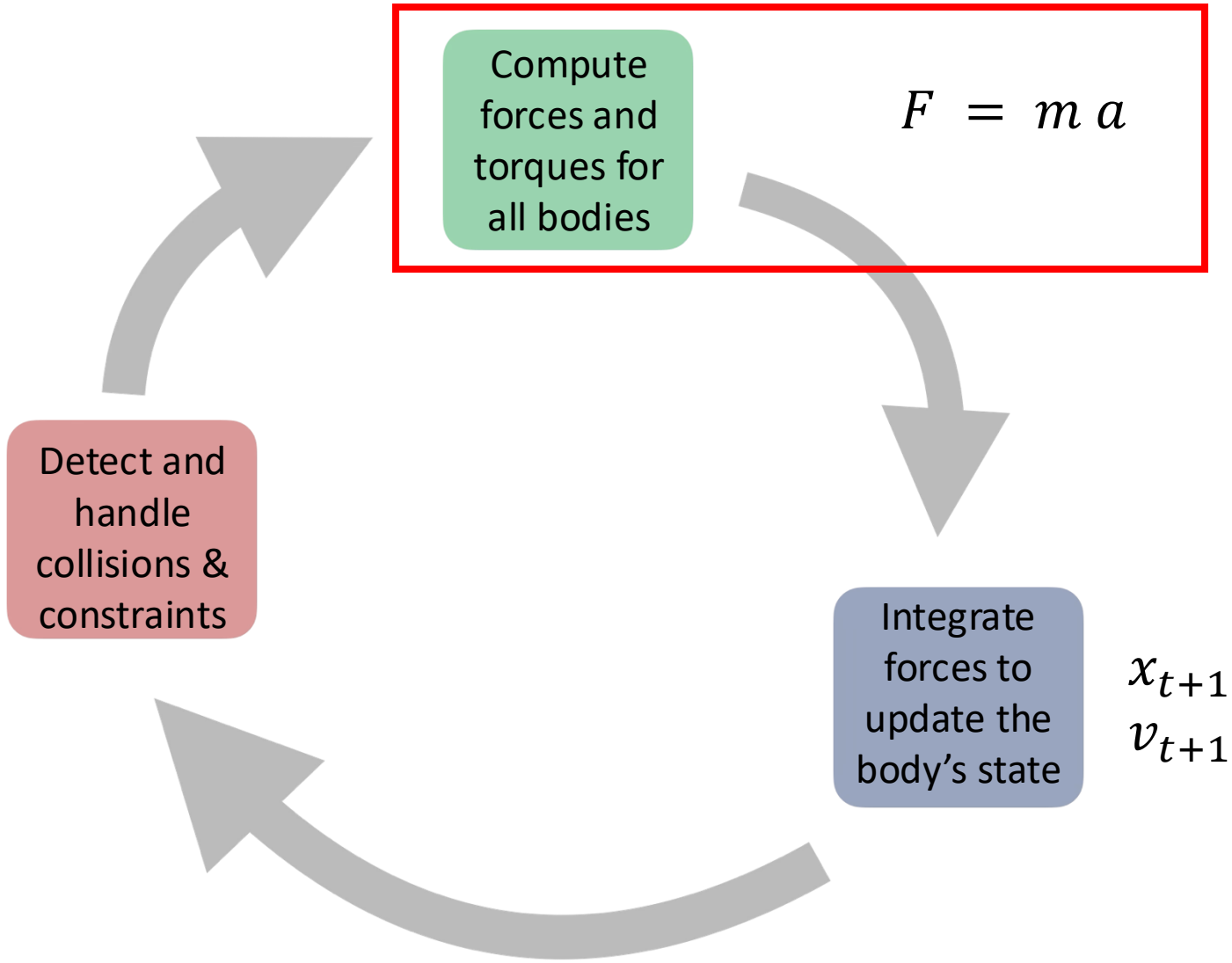

How do simulators handle collisions and joints?

- Jointly solve for contact impulses and generalized coordinates:

$$M(q)\ddot{q} = f(q, \dot{q}) + J(q)^T \lambda$$

- Each joint/collision adds **constraint equations** that can be linearized as:

$$J(q)\dot{q} = 0$$

Where:

- $J(q)$ is the joint/contact Jacobian mapping **generalized velocities** $\dot{q}$ to **constraint-space velocities** (e.g, relative velocity at a contact point).
- It equals to zero because we want the velocities to be zero during contact or constrained joint motion.

Discussion and Outlook

- Every simulator is built around the same core. What differs is the numerical sophistication and engineering effort behind each component:
  - Integrate forces
  - Handle collisions and constraints
  - Step forward in time

- **Limitations:**
  - Numerical drift and instability
  - Approximations in contact models
  - Non-differentiable behavior at collisions

- **Modern directions:**
  - Differentiable simulators
  - Learned physics models
  - GPU-accelerated massive simulation for training